Topology Optimization of Beam Supported at ends and Elastic Plate with Central Elliptical Hole using Optimality Criteria Approach in ANSYS

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ABSTRACT

Topological optimization is applied when design is at the conceptual level. In the present work, optimality criteria approach is implemented for the topology optimization of the 2-D beam structure which is supported by both ends and loaded vertically in the middle of its upper edge and elastic plate with a central elliptical hole is subjected to uniform longitudinal tensile stress $\boldsymbol{\sigma}_0$ at one end and clamped at the other end and analysis in ANSYS is done for quarter plate due to symmetry. Plane state of stress is assumed for the numerical examples considered.

In ANSYS use of the Solid Isotropic Material with Penalization (SIMP) method is done for the penalization scheme and the Optimality Criterion approach is used for topology optimization of the problem. The results of beam structure performed by the ANSYS based Optimality criterion are validated and compared with the results obtained by Bi-directional Evolutionary Structural Optimization (BESO) method.

Keywords -- Topology Optimization, Elastic Plate, Elliptic Hole, ANSYS, Pseudo-Densities, Compliance Minimization, Optimality Criterion and SIMP

1. INTRODUCTION

The paper presents the optimal design of the beam and elastic plate having elliptical hole at its centre. For given numerical problems the plane state of stress is considered. ANSYS is employed for carrying out topological optimization of the following structures. For example in the topology optimization of a beam structure, the discretization of the plate is done in small square elements where each element is controlled design variables which can vary continuously between 0 and 1. When a particular design variable has a value of 0, it is considered to be a hole, likewise, when a design variable has a value of 1, it is considered to be fully material. The elements with intermediate values are considered materials of intermediate densities.

The development of topological optimization can be attributed to Bendsøe and Kikuchi [1988], [1]. They presented a homogenization based optimization approach of topology optimization. The maximization of the integral stiffness of a structure composed of one or two isotropic materials of large stiffness using the homogenization technique was discussed by Thomsen [1992]. Numerical results are presented at the end of the paper.

It is well known that the solid-void topology optimization problem for continuum structures without a minimum size constraint generally lacks a solution. With the finite element analysis, the ESO method was initially proposed by gradually removing less efficient material with lower sensitivity numbers from the ground structure so that the remaining structure evolves towards an optimum (Xie and Steven 1993, 1997; Chu et al. 1996), [2]. It seems that the procedure coincides with finding a 0/1 solution by eliminating a feature smaller than one element. However, it is questionable when a lot of elements are removed simultaneously and never recovered because elemental sensitivity numbers are established at elemental level.

A later development in ESO is the introduction of Bidirectional ESO (BESO) where it allows elements to be added in the locations next to those elements with highest sensitivity numbers as well as to be removed in the region with lowest sensitivity numbers (Yang et al. 1999 [3]; Querin et al. 1998 [4]). However, this procedure is hard to control because there are two separate criteria for removing and adding elements. Therefore, we may get unsatisfactory results if the parameters are not set optimally (Rozvany 2008), [5]. More critical comments on various versions of ESO/BESO methods have been reviewed by Rozvany (Rozvany 2008; Tanskanen 2002 [6]; Edwards et al. 2007).

The results of SIMP with a filter scheme indicates that the solution would be convergent to a nearly 0/1 design if one chooses p sufficiently big. It provides the possibility of BESO to obtain similar solutions with two discrete variables.

One of the critical comments on the original ESO/BESO methods is that the procedure cannot be easily extended to other constraint, or multi-constraints problems (Rozvany 2008). Huang and Xie (2009b), [7] have demonstrated that the current BESO method can be extended to other constraints such as displacement. This paper will extend the current BESO method to the stiffness optimization with a material volume constraint and a local displacement constraint. The need for such constraints often comes from the technological background of the problem where the displacement at a certain node, not under load, is desired to lie within a prescribed value.

A web-based interface for a topology optimization program was presented by Tcherniak and Sigmund [2001]. The program is available over World Wide Web. The paper discusses implementation issues and educational aspects as well as statistics and experience with the program. Allaire et al. [2002] studied a level-set method for numerical shape optimization of elastic structures. The approach combines the level-set algorithm of Osher and Sethian with the classical shape gradient. Although this method is not specifically designed for topology optimization, it can easily handle topology changes for a very large class of objective functions. Rahmatalla and Swan [2004] presented a node-based design variable implementation for continuum structural topology optimization in a finite element framework and explored its properties in the context of solving a number of different design examples.

Sigmund and Clausen [2007] derived an approach to solve pressure load problems in topology optimization. Using a mixed displacement–pressure formulation for the underlying finite element problem, we define the void phase to be an incompressible hydrostatic fluid. Rozvany [2008] evaluated and compared the established numerical methods of structural topology optimization that have reached the stage of application in industrial software. Dadalau et al. [2008] presented a new penalization scheme for the SIMP method. One advantage of the present method is the linear density-stiffness relationship which has advantage for self weight or Eigen frequency problem. The topology optimization problem is solved through derived Optimality criterion method (OC), which is also introduced in the paper. Gunwant et al. obtained topologically optimal configuration of sheet metal brackets using Optimality Criterion approach through commercially available finite element solver ANSYS and obtained compliance versus iterations plots for various aspect ratio structures (brackets) under different boundary conditions.

Chaudhuri [8] worked on stress concentration around a part through hole weakening a laminated plate by finite element method. Peterson [9] has developed good theory and charts on the basis of mathematical analysis and presented excellent methodology in graphical form for evaluation of stress concentration factors in isotropic plates under in-plane loading with different types of abrupt change, but no results are presented for transverse loading. Patle et. al.[10] determined stress concentration factors in plate with oblique hole using FEM. Various angle of holes have been considered to evaluate stress concentration factors at such holes. The stress concentration factors are based on gross area of the plate.

The goal of topological optimization is to find the best use of material for a body such that an objective criterion (i.e. global stiffness, natural frequency, etc.) attains a maximum or minimum value subject to given constraints (i.e. volume reduction).

In this work, maximization of static stiffness has been considered. This can also be stated as the problem of minimization of compliance of the structure. Compliance is a form of work done on the structure by the applied load. Lesser compliance means lesser work is done by the load on the structure, which results in lesser energy is stored in the structure which in turn, means that the structure is stiffer.

ANSYS employs gradient based methods of topology optimization, in which the design variables are continuous in nature and not discrete. These types of methods require a penalization scheme for evolving true, material and void topologies. SIMP (Solid Isotropic Material with Penalization) is a most commonly penalization scheme, and is explained in the next section.

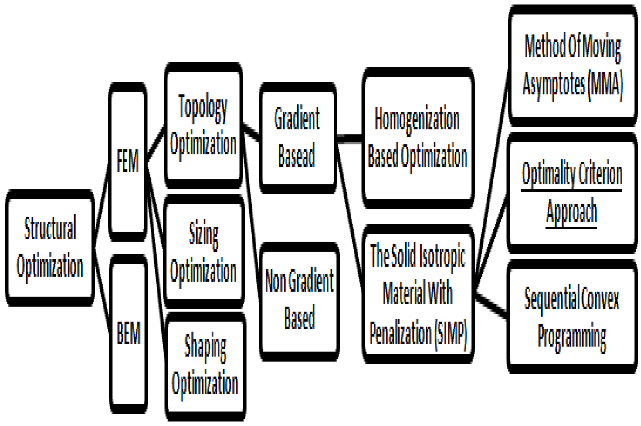

Fig -1: Layout of structural optimization

2. MATERIALS AND METHODS

The topology optimization is performed using optimality criteria method through ANSYS software. There are many approaches derived to solve pressure load problems in topology optimization. Structural analysis is used to assess the behaviour of engineering structures under the application of various loading conditions. Commonly used structural analysis method includes analytical methods, experimental methods and numerical methods.

Analytical method provides accurate solutions with applications limited to simple geometries. Experimental methods are used to test prototypes or full scale models. However they are costly and may not be feasible in certain cases. Numerical methods are most sought-after technique for engineering analysis which can treat complex geometries also. Among many numerical methods, finite element analysis (FEM) is the most versatile and comprehensive numerical technique in the hands of engineers today.

This process leads to a set of linear algebraic simultaneous equations for the entire system that can be solved to yield the required field variable (e.g., strains and stresses). As the actual model is replaced by a set of finite elements, this method gives an approximate solution rather than exact solution. However the solution can be improved by using more elements to represent the model.

2.1 The Optimality Criterion approach

As discussed earlier that the optimal design of the problems is performed using ANSYS which is based on optimality criterion approach. The discrete topology optimization problem is characterized by a large number of design variables, N in this case. It is therefore common to use iterative optimization techniques to solve this problem, e.g. the method of moving asymptotes, optimality criteria (OC) method, to name two. Here we choose the latter. At each iteration of the OC method, the design variables are updated using a heuristic scheme.

Based on these expressions, the design variables are updated as follows:

The Lagrange multiplier for the volume constraint Λ is determined at OC iteration using a bisection algorithm x_j. is the value of the density variable at each iteration step. u_j is the displacement field at each iteration step determined from the equilibrium equations.

The optimization algorithm structure is explained in the following steps:

- Make initial design, e. g. homogenous distribution of material.

-For this distribution of density, compute by finite element method the resulting displacements and strains.

-Compute the compliance of the design. If only marginal improvement in compliance over last design, stop iterations. Else, continue.

-Compute the update of design variable, this step also consists of an inner iteration loop for finding the value of Lagrange multiplier for the volume constraint.

-Repeat the iteration loop.

This paper considers the maximization of static stiffness through the inbuilt topological optimisation capabilities of the commercially available FEA software to search for the optimum material distribution in two plane stress structures. The optimum material distribution depends upon the configuration of the initial design space and the boundary conditions (loads and constraints). The goal of the paper is to minimize the compliance of the structure while satisfying the constraint on the volume of the material reduction. Minimizing the compliance means a proportional increase in the stiffness of the material. A volume constraint is applied to the optimisation problem, which acts as an opposing constraint. To visualize, more the volume of material, lower will be the compliance of the structure and higher will be the structural stiffness of the structure. For implementation of this, APDL codes for various beam modelling and topological optimisation were written and run in ANSYS.

2.2 Specimen Geometry and Boundary Conditions

In the present investigation, two specimen geometries and boundary conditions applied have been used as shown in the figures below. The specimen 1 is taken from the research paper of X. Huang · Y. M. Xie [2010], (Received: 22 August 2008 / Revised: 26 December 2008 / Accepted: 19 March 2009 / Published online: 9 April 2009 © Springer-Verlag 2009). Both the models are under plane state of stress.

The following numerical problems are considered as the linear elastic structures under plane state of stress conditions, point load in beam structure and longitudinal tensile stress in elastic plate having elliptical hole in its centre.

2.2.1 Model 1: Example 1 is a stiffness topology optimization problem for a beam structure which is supported by both ends and vertically loaded (P = 100 N) in the middle of its upper edge as depicted in Fig.1. The computations are performed in the domain with 200 × 100 four-node plane stress elements. The material is assumed with Young's modulus E = 1 GPa, Poisson's ratio ν = 0.3. The volume constraint is 30% of the design domain. The other parameters used for the following simulations are x_{min} = 0.001, p = 3, ER = 2% and r_{min} = 1.5 mm. The optimal topology without any displacement constraint is shown in Fig. 4 from the BESO method and in Fig. 5 the optimal design for the given problem using optimality criteria approach through ANSYS software package. Its mean compliance is 191 Nmm by BESO method and the use of OC approach is discussed in the results section (3.2).

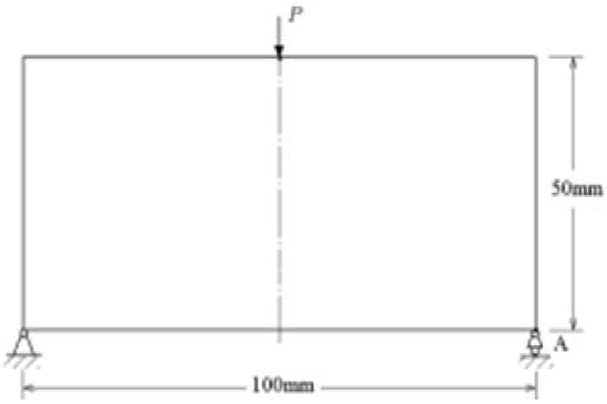

Fig-2: Design domain and loading and boundary conditions of Model 1

2.2.2 Model 2: In structure 2, the topology optimization of a central elliptical hole on the stress distribution and deflection in a rectangular plate of dimensions 400mm x 100mm x 10mm under longitudinal static load of magnitude 10MPa has been analysed using optimality criterion approach in ANSYS. Due to the presence of elliptical hole in the centre of plate, the maximum equivalent Von-Mises stress induced is expected at the corner of major axis of the hole. Due to the symmetry of the problem about the centre we are taking only a quarter plate for optimization in ANSYS. The Young's modulus (E) and Poisson's Ratio (ν) of the steel plate are taken to be equal to 2.1x10^5 N/mm^2 and 0.3 respectively.

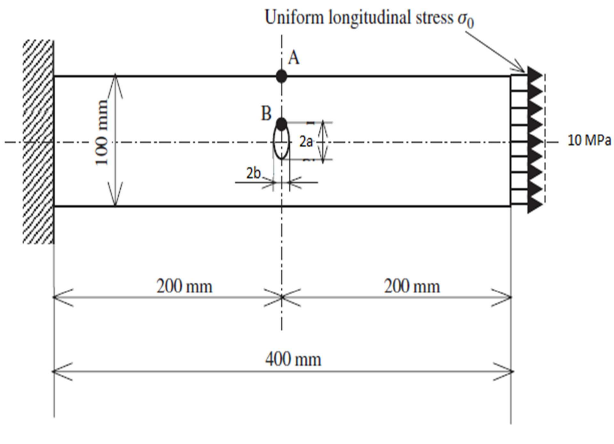

Fig-3: Geometry and boundary conditions of elastic plate with central elliptical hole

3. RESULTS

In this section the optimal topology of structures 1 and 2 is shown obtained from the Optimality Criteria Approach through ANSYS. Further the initial and final values of compliances for both the structures are shown in the charts[1 and 2]. Chart shows the graph between Compliance and iterations.

3.1 Structure Compared:

In this section, final compliance and optimal shape of the model 1(i.e. beam structure) obtained with the help of ANSYS based Optimality Criterion has been compared with a BESO method based a beam structure which is supported by both ends and vertically loaded (P = 100 N) in the middle of its upper edge.

3.2 Optimized Shape:

Figure 4, Shows the topology optimization through Bidirectional ESO Method which is nearly same as the topologically optimized shape as obtained for the beam structure under the given boundary conditions is obtained by using optimality criteria using ANSYS. Figure 5, shows the topologically optimized shape through ANSYS.

Fig.4: Optimal design for Model 1 by BESO method

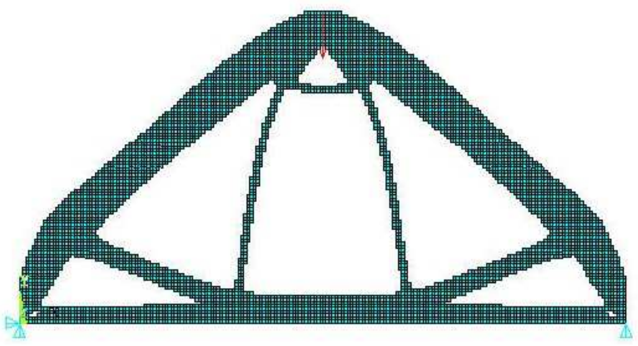

Fig-5: Optimal design for Model 1 using optimality criteria approach

The topologically optimized shape as obtained for the flat plate structure with a central elliptical hole under the given boundary conditions is obtained by using optimality criteria using ANSYS. Figure shows the topologically optimized shape.

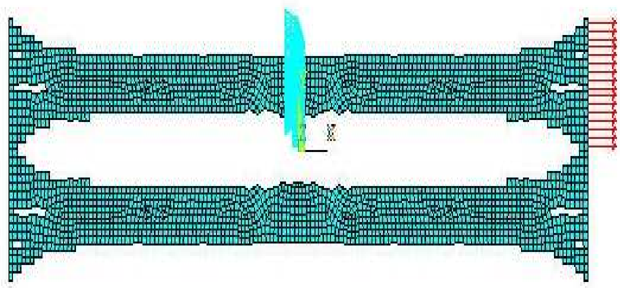

Fig-6: Optimal design for Model 2 using optimality criteria approach

3.3 Compliance:

For structure 1, the initial value of compliance was 966.76 Nmm and the final value as obtained after 31 iterations is 184.5 Nmm. A reduction of 597.76 Nmm from its initial value. Variation of compliance with iteration is shown in the graph below. Vertical axis represents the compliance and the horizontal axis represents the iteration.

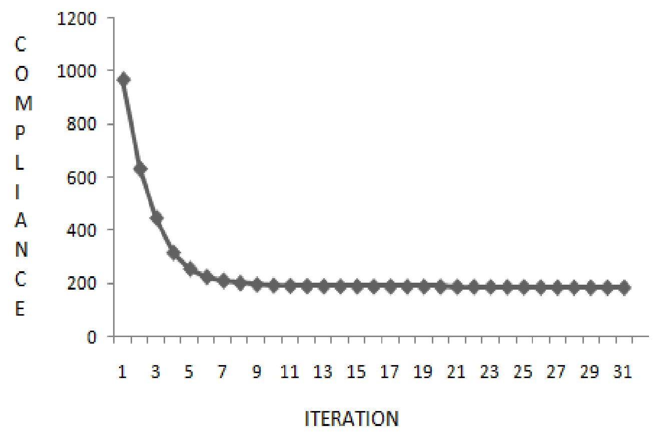

Fig-7: Compliance and iteration plot for beam structure (Model 1)

For structure 2, the initial value of compliance was 15.697 Nmm and the final value as obtained after 25 iterations is 10.231 Nmm. A reduction of 5.466 Nmm from its initial value. Variation of compliance with iteration is shown in the graph below. Vertical axis represents the compliance and the horizontal axis represents the iteration.

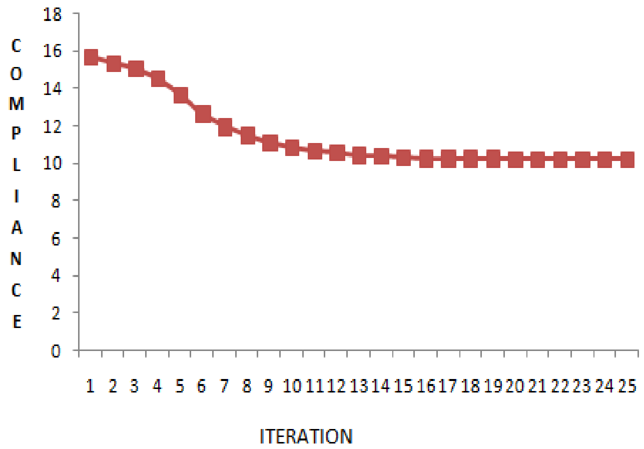

Fig-8: Compliance and iteration plot for central elliptical hole flat plate (Model 2)

The compliance obtained by ANSYS is nearly same as that obtained by BESO method.

3.3.1 For structure 1:

Compliance obtained by BESO method= 191Nmm.

Compliance obtained by ANSYS using optimality criteria method = 184.5 Nmm

Variation in two results= 6.5

The optimized shape obtained by optimality criteria using ANSYS is nearly same as that by BESO method.

3.3.2 For structure 2:

Compliance obtained by ANSYS using optimality criteria method = 10.231 Nmm

The optimized shape obtained for the elastic plate with a central elliptical hole by optimality criteria using ANSYS.

Table -1: Properties of Structures and final value of Compliances

S.No.	Structure	E	υ	Compliances
1.	Beam Supported at ends	1 GPa	0.3	184.5
2.	Plate with a central elliptical hole	2.1x10^5 N/mm^2	0.3	10.231

The above table shows the Young's modulus (E) and Poisson's ratio (υ) & final compliances value for structures optimized through ANSYS.

3. CONCLUSIONS

The optimized shape of model 1 using optimality criteria in ANSYS is nearly the same as that by the BESO method of topological optimization. Further the variation in compliance is very small. Also the compliance obtained from optimality criteria using ANSYS is less than that obtained by the BESO method, which is our basic objective of topological optimization. Thus ANSYS is an effective tool for topological optimization and the results obtained by ANSYS are more effective than the result obtained by the other method taken for comparison in this paper. For further work topology optimization of flat plate with an elliptical hole has been done.

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