ON TERNARY QUADRATIC DIOPHANTINE EQUATION

$z^2 = 50x^2 + y^2$

M.A.Gopalan[1],K.Geetha [2] and Manju Somanath[3]

1. Professor,Dept.of Mathematics,Shrimati Indira Gandhi College,Trichy- 02 Tamilnadu,India; e-mail: mayilgopalan@gmail.com
2. Asst Professor,Dept.of Mathematics,Cauvery College for Women,Trichy-18, Tamilnadu, India; e-mail:geetha_bothana@yahoo.co.in
3. Assistant Professor, Dept.of Mathematics, National College,Trichy- 01,Tamilnadu, India; e-mail:manjuajil@yahoo.com

ABSTRACT

The ternary quadratic diophantine equation $z^2 = 50x^2 + y^2$ is analyzed for its non-zero distinct integral points on it. A few interesting properties among the solutions are presented.

Key words: Integral points, Ternary quadratic, Polygonal numbers, Pyramidal numbers and Special numbers.

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Notation:

$t_{m,n}$ = Polygonal number of rank n with sides m

p_m^n = Pyramidal number of rank n with sides m

$ct_{m,n}$ = Centered Polygonal number of rank n with sides m

p_n = Pronic number

g_n = Gnomonic number

1. INTRODUCTION

Diophantine equations is an interesting concept, as it can be seen from [1-2]. For an extensive review of various problems one may refer [3-11]. In this context one may also see [12-23]. This communication concerns with yet another interesting ternary quadratic diophantine equation $z^2 = 50x^2 + y^2$ for determining its infinitely many non-zero integral solutions. Also a few interesting properties among the solutions are presented.

2. METHOD OF ANALYSIS

The ternary quadratic equation to be solved for its non-zero Integral solution is

$$z^2 = 50x^2 + y^2 \quad (1)$$

Assume

$$z = z(a,b) = a^2 + 50b^2 \quad (2)$$

where a,b are non-zero integer.

Different patterns of solutions for (1) are given below

PATTERN:1

On using equation (2) in (1) ,we get

$$50x^2 + y^2 = \left(a^2 + 50b^2\right)^2 \qquad (3)$$

On employing the method of factorization and comparing rational and irrational parts, we get,

$$\left(y + i\sqrt{50}x\right)\left(y - i\sqrt{50}x\right) = \left(a + i\sqrt{50}b\right)^2\left(a - i\sqrt{50}b\right)^2$$

On comparing the real and imaginary parts on the above equation, we get

$$\left.\begin{aligned} x &= x(a,b) = 2ab \\ y &= y(a,b) = a^2 - 50b^2 \end{aligned}\right\} \qquad (4)$$

Thus (2) and (4) represents non-zero distinct integral solutions of (1).

PROPERTIES:

1. $y(2n,b) + x(1,-2n) + z(2n,b) = 4t_{6,n}$
2. $x(1,-6n) + x(1,-2n^2) + y(3n,\mathrm{b}) + z(3n,\mathrm{b}) = 2t_{16,n}$

PATTERN:2

Equation (1) can be written as,

$$50X^2 + Y^2 = Z^2 * 1$$

Assume,

$$z = z(a,b) = a^2 + 50b^2$$

where a,b are non-zero distinct integers

write 1 as,

$$1 = \frac{\left(5 + i2\sqrt{50}\right)\left(5 - i2\sqrt{50}\right)}{225}$$

Using(6) and (7) in (5), we get

$$50x^2 + y^2 = \frac{\left(5 + i2\sqrt{50}\right)\left(5 - i2\sqrt{50}\right)}{225}\left(a^2 + 50b^2\right)^2$$

By the method of factorization and comparing the rational and irrational parts from the above equation, we get

$$x = x(a,b) = \frac{1}{15}\left[2a^2 - 100b^2 + 10ab\right]$$

$$y = y(a,b) = \frac{1}{15}\left[5a^2 - 250b^2 - 200ab\right]$$

As our interest is to find only integer solution,it is seen x and y are integer for suitable choices of a and b.

Let us assume, $a = 15A$ and $b = 15B$ the corresponding non-zero distinct integer solutions of (1) are found to be,

$$x = x(A,B) = 30A^2 - 1500B^2 + 150AB$$

$$y = y(A,B) = 75A^2 - 3750B^2 - 3000AB$$

$$z = z(A,B) = 225A^2 + 11250B^2$$

PROPERTIES

1. $y(A,1) - x(A,1) - 3\left(t_{32,A} - 518g_A + 1268\right) = 0$
2. $y(7,B) = -1875ct_{4,n} - 8625g_B + 3075$ (5)

REMARK:

It is worth mentioning here that 1 can also be represented as follws (6)

$$1)\ 1 = \frac{\left(6\sqrt{50} + i7\right)\left(6\sqrt{50} - i7\right)}{1849}$$

$$2)\ 1 = \frac{\left(12\sqrt{50} + i5\right)\left(12\sqrt{50} - i5\right)}{7225} \qquad (7)$$

PATTERN:3

Write (1) as,

$$(z + y)(z - y) = 50x^2$$

CASE:1

$$\frac{z + y}{50x} = \frac{x}{z - y} = \frac{P}{Q}$$

This is equivalent to the following two equations

$$-50xP + yQ + zQ = 0$$
$$xQ - Py - Pz = 0$$

$$\left.\begin{aligned} x &= x(P,Q) = -2PQ \\ y &= y(P,Q) = Q^2 - 50P^2 \\ z &= z(P,Q) = -Q^2 - 50P^2 \end{aligned}\right\} \quad (9)$$

Thus (9) represents non-zero distinct integral solutions which satisfy equation (1).

PROPERTIES:

1. $x(2^n, n) - y(1, 2^n) = -2wo_n - Mer_{2n} + 53$
2. $y(n, n+1) - z(n, n+1) - (2p_n + g_n + 3) = 0$

CASE:2

Equation (8)can also be rewritten as,

$$\frac{z+y}{x} = \frac{50x}{z-y} = \frac{P}{Q}$$

On following the procedure as in case (1) the non-zero distinct solutions of (1) are given by

$$x = x(P,Q) = -2PQ$$
$$y = y(P,Q) = 50Q^2 - P^2 \quad (10)$$
$$z = z(P,Q) = -50Q^2 - P^2$$

Thus (10) represents non-zero distinct integral solutions which satisfy equation (1).

PROPERTIES:

1. $x(p-1,1) + z(p-1,1) + 4t_{3,p} - 3g_p = 1$
2. $x(n+1, n-3) + t_{14,n} - t_{10,n} - g_n = 7$

PATTERN:4

Equation (1) can be written as,

$$z^2 - 50x^2 = y^2 * 1 \quad (11)$$

Write 1 as

$$1 = (\sqrt{50} + 7)(\sqrt{50} - 7) \quad (12)$$

Assume $y = a^2 - 50b^2$ (13)

Using(12) and (13) and in(11), we get

$$(z + \sqrt{50}x)(z - \sqrt{50}x) = (\sqrt{50} + 7)(\sqrt{50} - 7)(a^2 - 50b^2$$

Applying the method of cross multiplication,

On employing the method of factorization and equating the positive and negative factors, we get

$$x = x(a,b) = a^2 + 50b^2 + 14ab$$
$$y = y(a,b) = a^2 - 50b^2$$
$$z = z(a,b) = 7(a^2 + 50b^2) + 100ab$$

Thus (14) represents non-zero distinct integral solutions of (1).

PROPERTIES:

1. $y(2^n, n) = Mer_{2n} - 2t_{52,n} - 24g_n - 23$
2. $x(2a,1) + y(2a,1) = 2t_{10,a} + 17(g_a + 1)$

3. CONCLUSION

One may search for other patterns of solution and their corresponding properties.

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