Topological Optimization of Isotropic Material using Optimal Criteria Method

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ABSTRACT

Topology optimization mainly comprises of a mathematical approach that optimizes the layout within a given design constraints, for a given set of loads and boundary condition such that the performance matches with the prescribed set of performance targets. Topological optimization solve the problem of distributing a given amount of material in a design domain subjected to load and supports conditions, such that the compliance of the structure is minimized while the stiffness of structure is maximized. In topology optimization several approaches based on a density-like function were proposed, but resulted in optimization models with rather large number of design variable. An attractive alternative is optimal criteria method. This paper represents the optimal criteria method for topological optimization of isotropic material under different loads and boundary conditions with the objective to reduce mass of an existing material and study the different shape obtain by varying the mesh density of a structure. This paper work represents topological optimization for static loading using finite element solver ANSYS. APDL (ANSYS Parametric Design Language) has been employed for utilizing the topological optimization capabilities of commonly used finite element solver ANSYS. 8 node 82 elements are used to model and mesh the isotropic material in ANSYS.

Keywords-Topology Optimization, Pseudo-densities, Compliance minimization, Optimality Criterion, SIMP.

1. INTRODUCTION

The objective of the optimization problem is often some sort of maximization or minimization, for example minimization of compliance or maximization of stiffness. Mathematically the general optimization problem is most often formulated as minimization of the function subject to constraints, this can be expressed as

$$\text{Find } x \left\{ \begin{matrix} x_1 \\ x_2 \\ . \\ x_n \end{matrix} \right\} \text{ which minimize } f(x) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{subject to} \left\{ \begin{matrix} gi(x)\text{-}0,\ i=1, 2, \ldots, m \\ \\ hi(x)=0,\ i=1, 2, \ldots, n \end{matrix} \right. \quad \ldots\ldots\ldots\ldots(2)$$

Where x is the vector of design parameters and f(x) is the cost function. The functions $g_i(x)$ and $h_j(x)$ are called the inequality constraint function and the equality constraint function respectively and they define the constraints of the problem.

In a given design domain the purpose is to find the optimum distribution of material and voids. To solve this problem it is discretized by using the finite element method (FEM) and dividing the design domain into discrete elements (mesh). The resulting problem is then solved using optimization methods to find which elements that are material and which are not. This result in a so called 0-1 problem, the elements either exists or not, which is an integer problem with two different states for each element.

In topological optimization the design domain is created by assembling a large number of basic elements or building blocks. By beginning with a set of building block representing the maximum allowable region (region in space which the structure may occupy) each block is allowed to either exist or vanish from the design domain, a unique design is evolved. For example in the topology optimization of a cantilever plate, the plate is discretized into small rectangular elements (building blocks), where each element is controlled by design variables which can vary continuously between 0 and 1. When a particular design variable has a value of 0, it is considered to be a hole, likewise, when a design variable has a value of 1, it is considered to be fully material. The elements with intermediate values are considered materials of intermediate densities.

2. THE SIMP METHOD

The SIMP stands for Solid Isotropic Material with Penalization method. This is the penalization scheme or the power law approach, which is the basis for evolution of a 0-1 topology in gradient based methods. The power-law approach is physically permissible as long as simple conditions on the power are satisfied (e.g. p>3 for Poisson's ratio equal to 1/3)**.** The common choice of design parameterization is to take x_i as the design variable by convention, $x_i = 1$ at a point signifies a material region while $x_i = 0$ represents void. Each finite element (formed due to meshing in ANSYS) is given an additional property of pseudo-density, x_i where $0 \leq x_i \geq 1$, which alters the stiffness properties of the material.

$$x_i = \frac{\rho_i}{\rho_o} \text{ ………………..………………………..(3)}$$

Where,

ρ_i= Density of the i^th^ element

ρ_0= Density of the base material

x_i= Pseudo-density of the i^th^ element

This Pseudo-density of each finite element serves as the design variables for the topology optimization problem and the intermediate values are penalized according to the following scheme:

$$E_i = x_i^p E_o \text{ ……………………………….(4)}$$

Here E_i is the material young modulus of the i^th^ element while E^0 denotes the young modulus of the solid phase material. The stiffness of intermediate densities is penalized through the power law relation, so they are not favored. As a result, the final design consists primarily of solid and void regions.

$$K=K(x_i) =\Sigma EK_i=\Sigma x_i^p E^0 K_i \text{………………….. (5)}$$

3. MATERIAL AND METHOD

3.1 Optimal Criteria Approach

In topology optimization several approaches based on a density-like function were proposed, but resulted in optimization models with rather large number of design variables. Non-linear mathematic programming for such problems, on the other hand, is costly and time consuming. An attractive alternative is the optimality criteria method, which solve the optimality conditions directly. Two types of problems exist in the topological optimization. One is to minimize a performance function, subject to equilibrium equations and the constraint on the material resource. The other is to minimize the material resource, subject to equilibrium equations and performance functions.

The design region is meshed into a fixed grid of *n* finite elements. All elements carry densities that constitute the design variables. The objective is to find an optimal material distribution in the design domain that subjected to some given constraints, leading to minimizing a specified objective function. The standard approach is to let the design variables represent the relative densities of the material in related elements. To avoid the singularity of the matrix, the density variables are given a lower limit. Topology optimization problem is to minimize the compliance of the structure while it is subjected to a limited amount of material in the design domain can be written as

$$\begin{cases} \text{Minimize : } C(X) = \{F\}^T\{U\} \\ X = (x_1, x_2, \ldots, x_n)^T \\ \text{Subject to: } \begin{cases} V^T x \le V^* \\ 0 < x_{min} \le x_i \le 1 \\ \{F\} = [K]\{U\} \end{cases} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

where X is the design variable, C is the compliance of the structure, V^T is a vector containing the volume of the elements, V* is the volume constraint, K is the stiffness matrix.

Iterative optimization techniques For discrete topology optimization problem common to use to solve this problem, e.g. optimality criteria (OC) method. The Lagrangian function for the optimization problem is defined as:

$$\Gamma(x_i) = [U]^T[K][U] + \Lambda(\sum V^T - V^*) + \lambda_1([K][U] - [F]) + \sum \lambda_2(x_{\min} - x_i) + \sum \lambda_3(x_i - 1) \quad \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Where Λ, λ_1, λ_2 and λ_3 are Lagrange multipliers for the various constraints. The optimality condition is given by:

$$\frac{d\Gamma}{dx_i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(8)$$

Now, Compliance

$$C = \{U\}^T[K]\{U\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Differentiating Eq.(9) w. r. t. x_i, the optimality condition can be written as:

$$B_i = \frac{-\frac{dC}{dx_i}}{\Lambda V_i} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

Based on these expressions, the design variables are updated as follows:

$$x^{new} = \begin{cases} max(x_{min}, x_i - m) \\ \text{if } x_i B_i^{\eta} \le max(x_{min}, x_i - m), \\ x_i B_i^{\eta} \\ \text{if } max(x_{min}, x_i - m) < x_i B_i^{\eta} < min(1, x_i + m) \\ min(1, x_i + m) \\ \text{if } min(1, x_i + m) \le x_i B_i^{\eta} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Where, m is called the move limit and represents the maximum allowable change in x_i in a single OC iteration. Also, η is a numerical damping coefficient, and is usually taken to be 1/2. The Lagrange multiplier for the volume constraint Λ is determined at OC iteration using a bisection algorithm x_i is the value of the density variable at each iteration step u_i is the displacement field at each iteration step determined from the equilibrium equations.

3.2 Numerical Examples with Boundary Conditions

Three numerical examples are given to demonstrate the validity and efficiency of the proposed approach. The specimens are taken from the work of **Yiqiang Wang, Zhan Kang, Qizhi He [2013, 2014].** All the models are under plane state of stress. In first model the Young's modulus is E0 =100 while E0 = 1000.0 is taken in 2nd and 3rd. Poisson's ratio is μ= 0.3 is taken in all the models.

Table- I

1st model	Eo=100	μ= 0.3
2nd model	Eo=1000	μ= 0.3
3rd model	Eo=1000	μ= 0.3

Table-1: Material Properties used

MODEL-1 Topology optimization of a cantilever beam with a fixed circular hole

The first example focuses on the cantilever beam with a fixed hole as shown in fig-1. The center of the circle locates at (17.5, 15), with a radius R_{hole} = 7.

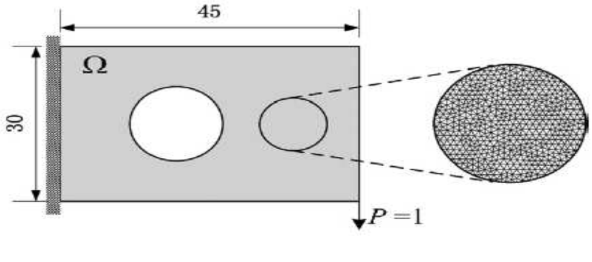


Fig-1: cantilever beam with a fixed circular hole

MODEL-2 Topology optimization of a bracket with two holes

To illustrate the effectiveness of the proposed optimization approach more complex geometries, the second example considers the topological design of a bracket structure in a design domain with two same-sized circular holes, as shown in fig-2. The left circular hole is fixed while a uniformly distributed line forces t = 1/pi*r is applied to the left-half boundary of the right hole. The volume fraction is given by fv = 0.35.

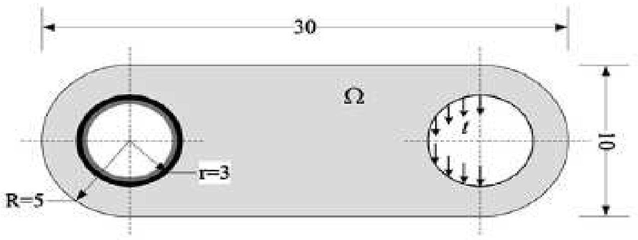


Fig-2: bracket with two holes

MODEL-3 Topology optimization with a half-ring shaped design domain

The final example refers to the optimal topology design within a half-ring shaped design domain, whose geometrical dimensions and boundary conditions are schematically plotted in fig-3. The volume fraction is set to be fv = 0.5.

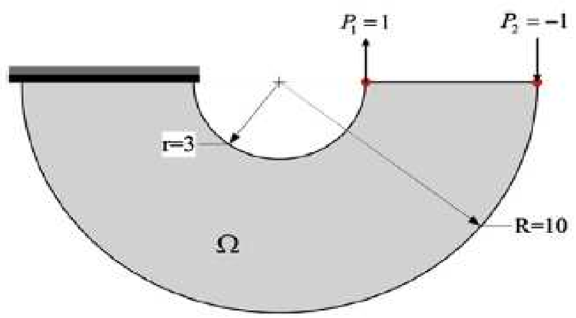


Fig-3: half-ring shaped design domain

4. RESULTS AND DISCUSSION

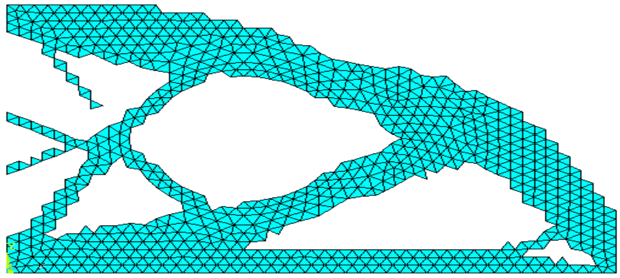
This section presents the detailed results of FE analysis and optimization of the above structures. Final compliance and optimal shape of the models obtained with the help of gradient based ANSYS based Optimality Criterion have been compared with an adaptive refinement approach in the work of Yiqiang Wang, Zhan Kang, Qizhi He [2013, 2014].

MODEL 1: In Topology optimization of a cantilever beam with a fixed circular hole meshing is done with 8 nodes 82 triangular element by giving element edge length one for each line. Table-II shows the final compliance obtained in the case of ANSYS based OC and adaptive refinement approach.

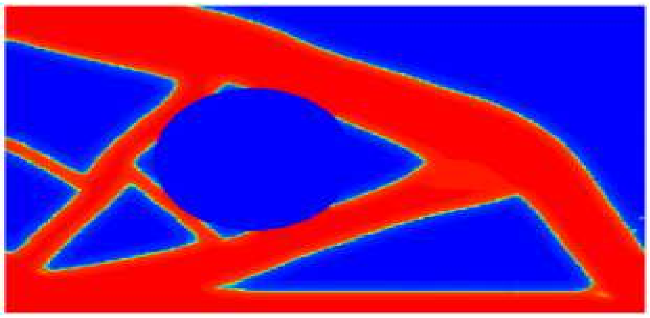
Table- II

Method	ANSYS based OC	adaptive refinement approach
Compliance	0.44176	0.471
Iteration	34	40
Percentage difference in compliance	2.924	

Table 2: Comparison for model 1



(a)



(b)

Fig-4: Optimal shapes obtained by (a) ANSYS based OC and (b) adaptive refinement approach

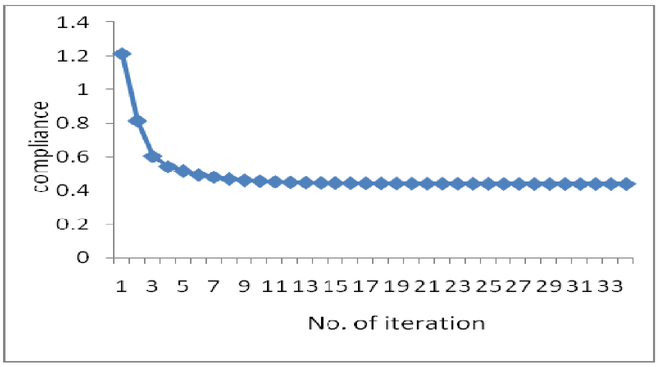


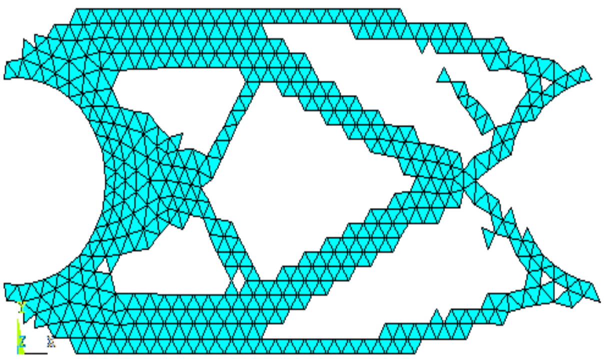
Fig-5: Graph between compliance and iteration for model 1

MODEL 2: In Topology optimization of a bracket with two holes meshing is done with 8 nodes 82 triangular element by giving element edge length 0.2 for each line and total number element is 1996. Table III shows the final compliance obtained in the case of ANSYS based OC and adaptive refinement approach.

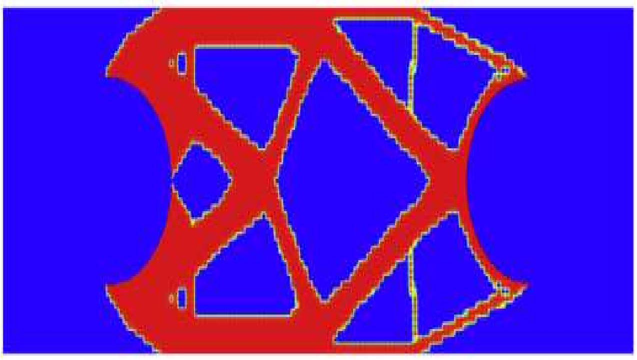
Table- III

Method	**ANSYS based OC**	**adaptive refinement approach**
Compliance	0.0363	0.045
Iteration	41	73
Percentage difference in compliance	23.97	

Table-3: Comparison for model 2



(a)



(b)

Fig-6: Optimal shapes obtained by (a) ANSYS based OC and (b) adaptive refinement approach

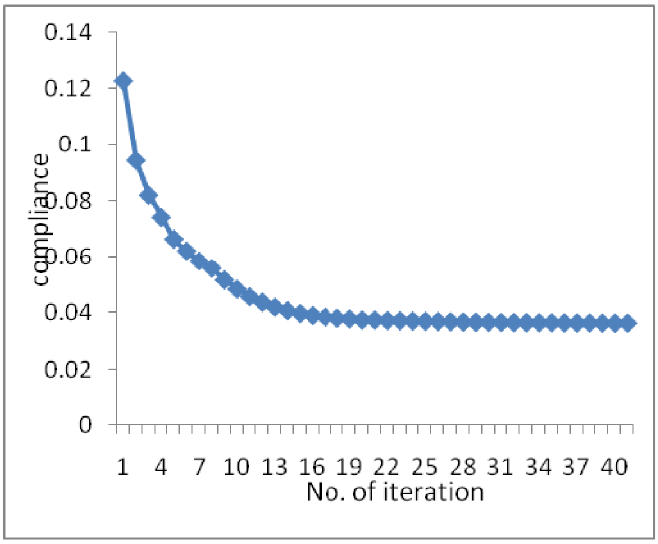


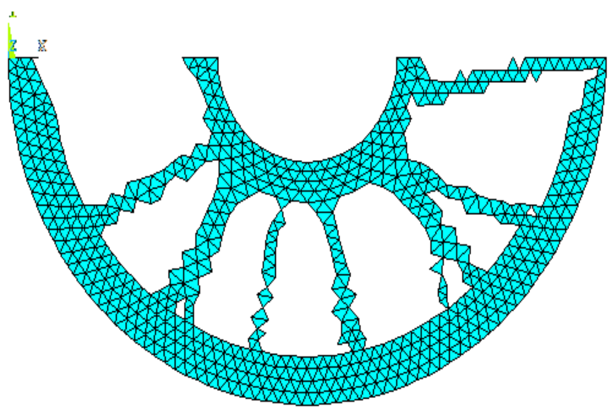
Fig-7: Graph between compliance and iteration for model 2

MODEL 3: In Topology optimization with a half-ring shaped design domain meshing is done with 8 nodes 82 triangular element by giving element edge length 0.6 for each line and total number element is 1996. Table IV shows the final compliance obtained in the case of ANSYS based OC and adaptive refinement approach.

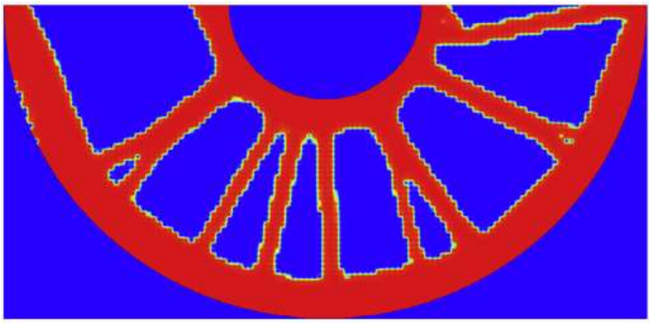
Table- IV

Method	**ANSYS based OC**	**adaptive refinement approach**
Compliance	0.05379	0.057
Iteration	33	64
Percentage difference in compliance	9.063	

Table-4: Comparison for model 4



(a)



(b)

Fig-8: Optimal shapes obtained by (a) ANSYS based OC and (b) adaptive refinement approach

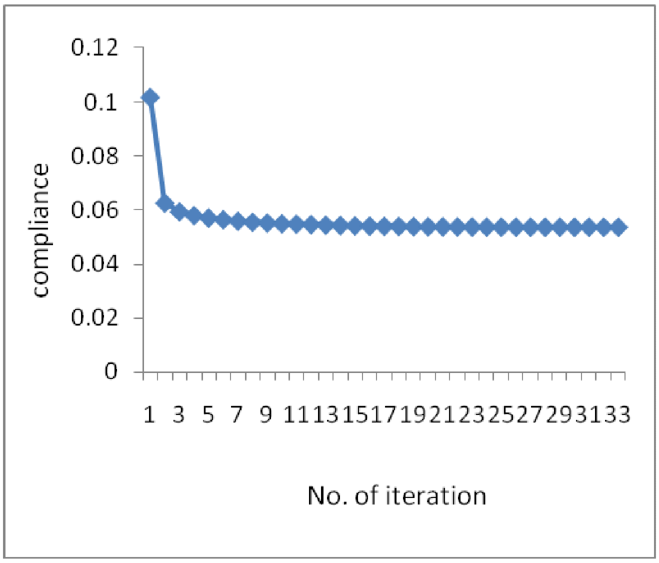


Fig-9: Graph between compliance and iteration for model 3

5. CONCLUSION

The modeling and finite element analysis has been presented for three different models. Topology optimization was performed and according to the results, it can be concluded that the results of ANSYS based Optimality Criterion which is a gradient based method are compared with those obtained by adaptive refinement approach. Compliance values obtained by ANSYS based Optimality Criterion are lower by **2.94%-23.97%** than adaptive refinement approach in the work of Yiqiang Wang, Zhan Kang, Qizhi He [2013, 2014]. On comparison it can also be conclude that number of iteration by ANSYS based Optimality Criterion are less than the adaptive refinement approach. Topology optimization being the primary stage of structural optimization, the above plane stress structures can be considered for shape optimization and sizing optimization. In shape optimization, the design variables can be considered to be the coordinates of the nodes and in sizing optimization, any physical dimensions. The objective variable in both cases can be the volume of the structure. Material optimization approach will be considered for future research.

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REFERENCES

[1] Allaire, G. , Jouve, F. and Toader, A. M. (2002) "A level set method for shape optimization" *C. R. Acad. Sci. Paris.*

[2] Bendsøe, M. P. and Kikuchi, N. (1988) "Generating optimal topologies in structural design using a homogenization method" *Comput. Meth. Appl. Mech. Eng*., vol: 71: 197-224.

[3] Chapman, C. D. (1994) "Structural topology optimization via the genetic algorithm", Thesis, M. S. Massachusetts Institute of Technology, America.

[4] Chiandussi, G. M. , Codegone and Ferrero, S." Topology optimization with optimality criteria and transmissible loads", Computers and Mathematics with Applications 57 (2009) 772_788

[5] Diaz, A. and Sigmund, O. (1995), "Checkerboard patterns in layout optimization" *Struct. Optim.*. Vol: 10: 40-45

[6] Michael, Thomas R. (2010) "Shape and topology optimization of brackets using level set method", An Engineering project submitted to the graduate faculty of Rensselaer Polytechnic Institute in partial fulfillment of the degree of Master of Engineering in Mechanical Engineering. Rensselaer Polytechnic Institute Hartford, Connecticut

[7] Rahmatalla, S. F. and Swan, C. C. (2004) "A Q4/Q4 continuum structural topology optimization implementation", *Struct. Multidisc. Optim. Springer-Verlag*, Vol 27: 130-135

[8] Rozvany, G. I. N. "A critical review of established methods of structural topology optimization", Struct Multidisc Optim (2007)

[9] Sigmund, O. and Petersson, J. (1998) “Numerical instabilities in topology optimization: A survey on procedures dealing with checkerboards, mesh-dependencies and local minima”, *Struct. Optim.*. Vol 16: 68-75

[10] Sigmund, O. "A 99 line topology optimization code written in Matlab’’ *Struct. Multidisc. Optim. Springer-Verlag* 2001, Vol 21: 120-127.

[11] Swan, C. C. and Kosaka, I. (1997) “Voigt-Reuss topology optimization for structures with linear elastic material behaviors”, Int. *J. Numer. Meth. In Eng*. Vol: 40: 3033-3057

[12] Tcherniak, D. and Sigmund, O. “A web-based topology optimization program” *Struct. Multidisc. Optim. Springer-Verlag* 2001, Vol 22: 179-187

[13] Wang, Yiqiang, Kang, Zhan and He, Qizhi “Adaptive topology optimization with independent error control for separated displacement and density fields” Computer and Structure 135 (2014) 50–61

[14] Wang, Yiqiang, Kang, Zhan and He, Qizhi “An adaptive refinement approach for topology optimization based on separated density field description” Computer and Structure 117 (2013) 10–22

